

Information and Learning in Markets

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Chapter 9

Price and Information Dynamics in Financial Markets

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Plan of the Chapter

In this chapter we look further at dynamic markets and include strategic traders. We will look at

- 1 Dynamic market order markets; herding and slow learning; speed of information revelation.
- 2 Trading with long-lived information (Kyle (1985) model) and extensions.
- 3 Market manipulation
- 4 Strategic trading when information is short-lived
- 5 Dynamic hedging strategies of large risk averse traders.

9.1 Sequential Trading, Market order Markets, and Speed of Learning

This section studies sequential trade models:

- Glosten and Milgrom (1985) and relate it to the results on herding.
- A model of learning from past prices – a variation of the Cournot-type model (Vives (1993)).
- A Price discovery mechanism (Vives (1995)).



9.1 Sequential Trading, Market order Markets, and Speed of Learning

9.1.1 Sequential Trading and Herding

Glosten and Milgrom (1985)

- sequential trading model with a risky asset with unknown liq. value θ .
- Competitive risk neutral market makers set a bid-ask spread.
- A single investor arrives each period and trades only once a single unit of the asset with market makers.
- The investor receives a private signal about the stock and posts an order which can be
 - Information motivated with prob. μ .
 - Liquidity motivated with prob. $1 - \mu$.
- The history of transactions (prices and quantities) is known at any period and the type of the trader is unknown to the market makers.
- In this setup:
 - The ask price is the conditional expectation of θ given a buy order and past public information.
 - The bid price is the conditional expectation of θ given a sell order and past public information.

9.1 Sequential Trading, Market order Markets, and Speed of Learning

9.1.1 Sequential Trading and Herding

- Adverse selection implies that there is a positive, and increasing in μ , spread.
- However, spreads will **not** be period-by-period larger in a market with a higher μ since a larger proportion of insiders implies
 - larger initial spread but
 - faster information revelation.
- Market makers on average lose money on informed trades, balancing these losses with the profits obtained from liquidity-motivated trades.
- The bid and ask price converges to the true value as market makers accumulate information.
- The role of the depth parameter λ in the competitive price formation models is played here by the bid-ask spread.
- Transaction prices follow a martingale.

9.1 Sequential Trading, Market order Markets, and Speed of Learning

9.1.1 Sequential Trading and Herding

Avery and Zemsky (1998)

- Even though the information structure similar to Bikhchandani et al. (1992), in which the informed investor receives a noisy signal about the value of the stock. . .
- an informational cascade and herding **will not** occur.
- Reason: the price is a continuous public signal that keeps track of aggregate public information.
- Suppose traders ignore their private signals,
 - ① The price cannot reveal any information.
 - ② Both the bid and the ask prices must equal the probability that the value is high given public information.
 - ③ Then, an informed trader has an incentive to follow his signal.
- With only two possible liquidation values there cannot be herding.

9.1 Sequential Trading, Market order Markets, and Speed of Learning

9.1.1 Sequential Trading and Herding

Still, Park and Sabourian (2006)

- Herding can arise with three possible states when there is enough noise and traders believe that extreme outcomes are more likely than intermediate ones.
- This may happen even if signals conform to a standard monotone likelihood ratio property.

Romer (1993)

- Herding and crashes arise naturally when traders are uncertain about the precision of information of other traders.
- In this case market makers may update the price little after observing the order flow because of the uncertainty on the quality of information in the market.

9.1 Sequential Trading, Market order Markets, and Speed of Learning

9.1.1 Sequential Trading and Herding

Another way to think about the result

- Competitive market making induces a payoff externality on informed traders.
- Investors, as well as market makers, learn from past trades. The changes in the bid-ask spread due to competitive market making (implying a payoff externality) offset the incentive to herd (because of the informational externality).
- As market makers learn more about the fundamental value the bid-ask spread is reduced and this entices an informed investor to use his information.

Dow (2004)

- extends the Glosten-Milgrom model to incorporate expected-utility maximizing liquidity traders and shows that multiple equilibria with different endogenous levels of liquidity may arise.
- Equilibria have the familiar bootstrap property: if a high (low) level of liquidity is anticipated, the liquidity traders increase (decrease) their trading intensity and the spread is small (large).

9.1 Sequential Trading, Market order Markets, and Speed of Learning

9.1.2 Slow Learning from Past Prices

Consider the following model where agents learn from past prices: ♣

- Informed traders are risk neutral but face a quadratic adjustment cost in their position.
- The horizon is infinite and at each period there is an independent (small) probability $1 - \delta > 0$ that the ex post liquidation value of the risky asset θ is realized.
- The probability of θ not being realized at period t , $\delta^t \rightarrow 0$, as $t \rightarrow \infty$.
- Each agent of a continuum of long-lived traders receives a private noisy signal about θ at $t = 1$ and submits a market order to a centralized market clearing mechanism.
- At period t the information set of agent i is $\{s_i, p^{t-1}\}$.
- Noise traders demand: $u_t - p_t$, where u_t is a random intercept that follows a white noise process.
- Δx_t : aggregate demand of the informed traders in period t .
- Market clearing condition in t : $u_t - p_t + \Delta x_t = 0$.

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9.1.2 Slow Learning from Past Prices

- From the quantity Δx_{it} demanded in period t , trader i obtains profits

$$\pi_{it} = (\theta - p_t)\Delta x_{it} - \lambda \frac{(\Delta x_{it})^2}{2}.$$

- Total profits associated with the final position $\sum_{k=1}^t \Delta x_{it}$ are $\sum_{k=1}^t \pi_{it}$.
- At any period an informed trader maximizes the (expected) discounted profits with discount factor δ .
- Traders do learn from past prices and public information eventually reveals θ but the speed of learning is slow (at the rate $1/\sqrt{t^{1/3}}$) if there is no positive mass of perfectly informed traders.
- In this case the asymptotic variance of public information in relation to θ is $(3\tau_u)^{-1/3}(\lambda/\tau_\epsilon)^{2/3}$ and it increases with the amount of noise trading, average noise in the signals, and the slope of adjustment costs.

9.1 Sequential Trading, Market order Markets, and Speed of Learning

9.1.3 Price Discovery, Speed of Learning, and Market Microstructure

“Slow learning” result depends on the market microstructure: market makers may accelerate the speed of learning and we recover the standard convergence rate. ♣

- Market with a single risky asset, with random ex post liquidation value θ , and a riskless asset, with unitary return.
- Continuum of risk-averse competitive informed agents and price sensitive noise traders. The profits of agent i with position x_i :
$$\pi_i = (\theta - p)x_i.$$
- Informed agents have CARA utilities $U(\pi_i) = -\exp\{-\rho\pi_i\}$, $\rho > 0$, and their initial wealth is normalized to zero.
- Informed agent i submits a market order contingent on his information.
- Noise traders submit in the aggregate a price-sensitive order $u - p$.

9.1 Sequential Trading, Market order Markets, and Speed of Learning

9.1.3 Price Discovery, Speed of Learning, and Market Microstructure

- Price discovery is modeled as an information tâtonnement with potentially many stages.
- At stage t

with prob. $\begin{cases} \gamma_t > 0 & \text{market opens, value is realized and trade occurs} \\ 1 - \gamma_t & \text{there is no trade and the tâtonnement continues.} \end{cases}$

- If at the beginning of stage t the market has not opened the competitive informed agents, before knowing whether there will be trade in the period, have the opportunity to place orders and noise traders place them.

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- These orders supersede previous orders, which are understood to be cancelled if the market does not open. The auctioneer or a centralized trading mechanism quotes a notional price and in the next round traders can revise their orders.
- Information tâtonnement processes are used in the preopening period of continuous, computerized, trading systems in several exchanges (e.g., Paris Bourse, Toronto Stock Exchange, Bolsa de Madrid, or the Arizona Stock Exchange (AZX)).

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9.1.3 Price Discovery, Speed of Learning, and Market Microstructure

The price discovery process works as follows.

- Traders submit orders to the system for a certain period of time before the opening (one hour or one hour and a half) and theoretical market clearing prices are quoted periodically as orders accumulate.
- No trade is made until the end of the tâtonnement and at any point agents may revise their orders. This preopening auction is designed to decrease the uncertainty about prices after a period without trade.
- In the Deutsche Börse with the Xetra system there is an opening auction which begins with a call phase in which traders can enter and/or modify or delete existing orders before the (short) price determination phase. The indicative auction price is displayed when orders are executable. The call phase has a random end after a minimum period.

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9.1.3 Price Discovery, Speed of Learning, and Market Microstructure

- The information tâtonnement process can be interpreted as a mechanism to elicit the aggregate information of informed agents via price quotations.
- It is analogous to the “dynamic information adjustment process” considered by Jordan (1982, 1985) to implement rational expectations equilibria.
- In both cases prices serve only as public information signals and trades are not realized until the iterative process has stopped.
- Similarly, Kobayashi (1977) assumes that agents trade at any period as if it were the last.

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9.1.3 Price Discovery, Speed of Learning, and Market Microstructure

Exogenous Market Depth

- Information set of trader i : $\{s_i, p^{t-1}\}$.
- The trader places a market order $X_t(s_i, p^{t-1})$ and noise traders submit the aggregate price contingent order $u_t - p_t$.
- Limit order book is:

$$L_t(p_t) = \omega_t - p_t, \text{ with } \omega_t = x_t + u_t, \text{ and } x_t = \int_0^1 x_{it} di.$$

- The auctioneer quotes a price $p_t = u_t + x_t$ to clear the market.
- As time evolves the depth of the market is fixed (at 1) and the price elastic noise traders avoid the market breaking down.

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9.1.3 Price Discovery, Speed of Learning, and Market Microstructure

All random variables are assumed to be normally distributed.

- The sequence $\{u_t\}$ is independently and identically distributed with zero mean and variance σ_u^2 .
- Private signals are given by $s_i = \theta + \epsilon_i$, where $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$ and $\epsilon_i \sim N(0, \sigma_\epsilon^2)$, with $\text{Cov}[\theta, u] = \text{Cov}[\theta, \epsilon_i] = \text{Cov}[\epsilon_i, u_t] = \text{Cov}[\epsilon_i, \epsilon_j] = 0$ $j \neq i$, for all t .
- At stage t a strategy for trader i is a function that maps his private information and the observed past prices p^{t-1} into desired trades.
- The asset is liquidated and trade realized in the period with probability γ_t and with probability $1 - \gamma_t$ the trader obtains the continuation (expected) utility which, because the agent's size is negligible, is independent of his market order in period t .
- Agents behave as if the asset were to be liquidated and trade realized in the period.

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9.1.3 Price Discovery, Speed of Learning, and Market Microstructure

- At period t the price sequence p^t can be summarized in a public statistic $\theta_t \equiv E[\theta|p^t] \equiv E[\theta|z^t]$, where $z_t = a_t\theta + u_t$, with a_t the response coefficient of traders to private information.
- In this market the price p_t is **not** a sufficient statistic for the information in the sequence of prices because there is no **competitive risk neutral market making** sector.
- Given the CARA utility function and the optimality of myopic behavior the demand of trader i will be given by

$$X_t(s_i, \theta_{t-1}) = \frac{E[\theta - p_t | s_i, \theta_{t-1}]}{\rho \text{Var}[\theta - p_t | s_i, \theta_{t-1}]}.$$

- Equilibrium strategies are symmetric and linear in s_i and θ_{t-1} , with .

$$a_t = \frac{\tau_\epsilon \tau_u (1 - a_t)}{\rho((1 - a_t)^2 \tau_u + \tau_\epsilon + \tau_{t-1})},$$

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- This yields a recursive cubic equation,

$$G_t(a) \equiv (1 - a)(\rho a \tau_\epsilon^{-1}(1 - a) - 1)\tau_u \tau_\epsilon + \rho a(\tau_\epsilon + \tau_{t-1}) = 0,$$

with potentially three solutions in the interval $(0, 1)$.

- For t large it can be checked that the solution is unique.
- The response to public information is given by

$$c_t = \frac{\tau_{t-1}}{a_t^{-1} \tau_\epsilon + (\tau_\epsilon + \tau_{t-1})(1 - a_t)^{-1}}.$$

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For any linear equilibrium, as $t \rightarrow \infty$

- $a_t \rightarrow 0$, $c_t \rightarrow (\rho\sigma_u^2 + 1)^{-1}$.
- Both a_t and the informativeness of the price statistic θ_t , $\tau_t = \text{Var}[\theta|\theta_t]^{-1}$ are of the order $t^{1/3}$.
- Asymptotic precision $\lim_{t \rightarrow \infty} t^{1/3}\tau_t \equiv A\tau_\infty = 3^{1/3}\tau_u(\tau_\epsilon/\rho)^{2/3}$.
- Thus, θ_t converges almost surely and in mean square to θ as $t \rightarrow \infty$ and

$$\sqrt{t^{1/3}}(\theta_t - \theta) \xrightarrow{L} N(0, ((\rho\sigma_\epsilon^2)^2/3)^{1/3}\sigma_u^2).$$

- Convergence to the shared-information equilibrium, where informed agents pool their information, learn θ , and trade an amount $X(\theta) = \theta(1 + \rho\sigma_u^2)^{-1}$, is at the slow rate $1/\sqrt{t^{1/3}}$.
- Slow convergence is due to the fact that a_t converges to zero as t increases since Bayesian agents decrease the weight they put on their private signals as public information becomes better and better.

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9.1.3 Price Discovery, Speed of Learning, and Market Microstructure

Endogenous Market Depth

Suppose a competitive, risk-neutral market making sector prices the asset:

- Competitive market makers set

$$p_t = E[\theta | \omega^t],$$

where $\omega^t = \{\omega_k\}_{k=1}^t$, and ω_t is the intercept of the order book
 $L_t(p_t) = \omega_t - p_t$.

- The current price p_t is now a sufficient statistic of all past and current prices p^t which in turn are observationally equivalent to ω^t :

$$E[\theta | p^t] = E[\theta | \omega^t] = p_t.$$

- Letting $\tau_t \equiv \text{Var}[\theta | p_t]^{-1}$ we have that $\text{Var}[p_t | p_{t-1}] = \tau_{t-1}^{-1} - \tau_t^{-1}$.

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Vives (1995) shows:

Proposition

With a competitive risk neutral market making sector there is a unique linear equilibrium. Traders use symmetric strategies

$$X_t(s_i, p^{t-1}) = a_t(s_i - p_{t-1}), \text{ where } a_t = (\rho(\sigma_\epsilon^2 + \text{Var}[p_t|p_{t-1}])))^{-1},$$

and prices are given by $p_t = \lambda_t \omega_t + p_{t-1}$, $\omega_t = a_t(\theta - p_{t-1}) + u_t$, with $\lambda_t = \tau_u a_t / \tau_t$, $\tau_t = \tau_\theta + \tau_u \sum_{k=1}^t a_k^2$, and $\text{Var}[p_t|p_{t-1}] = \tau_{t-1}^{-1} - \tau_t^{-1}$.

For a proof see the [Appendix](#) .

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Properties of the equilibrium:

- Trader i buys (sell) according to whether his private estimate of θ , s_i , is larger (smaller) than the market estimate, p_{t-1} .
- Informed traders' response to private information, a_t , depends negatively on ρ , σ_ϵ^2 and $\text{Var}[p_t|p_{t-1}]$.
- Informed traders optimize against the linear function $p_t = \lambda_t \omega_t + p_{t-1}$, and market makers determine the price function $p_t = E[\theta|\omega^t]$, making λ_t endogenous.
- An increase in the depth of the market induces risk averse informed traders to respond more to their information (since λ_t is lower).
- On the contrary, an increase in a_t , induces market makers to put more weight on the order flow in setting p_t , decreasing market depth, as the order flow is more informative.
- An increase in the precision of prices τ_t , holding a_t constant, has the opposite effect.

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Asymptotic properties of the linear equilibrium as $t \rightarrow \infty$:

- a_t converges monotonically from below to $(\rho\sigma_\epsilon^2)^{-1}$.
- τ_t and λ_t^{-1} tend to infinity at a rate of t .
- $\text{Var}[p_t]$ converges monotonically from below to σ_θ^2 .
- $\text{Var}[p_t|p_{t-1}]$ tends to 0.
- The expected volume traded by informed agents converges from above to $(2/\pi)^{1/2}(\rho\sigma_\epsilon)^{-1}$.
- The expected total volume traded converges from above to $(2/\pi)^{1/2}((\rho\sigma_\epsilon)^{-1} + 2\sigma_u)$.

From the fact that the precision of prices grows linearly with t :

- 1 p_t converges (almost surely and in mean square) to θ at a rate of $1/\sqrt{t}$.
- 2 $\sqrt{t}(p_t - \theta)$ converges in distribution to $N(0, \sigma_u^2 \rho^2 \sigma_\epsilon^4)$.

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In contrast to the exogenous depth market

- a_t increases with t and converges to a positive constant.
- Market depth is endogenous and increasing as more tâtonnement rounds accumulate because of market makers.
- A risk averse trader responds more to the deviations of p_{t-1} from the private signal s_i the deeper is the market.
- The new information in the current price $z_t = a_t\theta + u_t$, does not vanish for t large and the order of magnitude of the precision of prices τ_t is t .

Summarizing:

- With a competitive market making sector price quotations converge to θ fast, at the rate $1/\sqrt{t}$, and the asymptotic precision of p_t is decreasing in ρ , σ_u^2 and σ_ϵ^2 .
- Convergence is slower if agents are more risk averse, have less precise private information, or noise is larger.
- The precision of prices grows linearly with the number of rounds t .

9.1 Sequential Trading, Market order Markets, and Speed of Learning

9.1.4 Summary

This section has demonstrated that

- Market microstructure matters for the information revelation properties of prices.
- Market makers modify the depth of the market in response to the information content of the order flow and speed up information revelation by prices.
- In the Glosten and Milgrom (1985) model the continuous price variable avoids informational cascades and herding.
- In the Vives (1993 and 1995a) models market makers avoid a slow learning outcome. Indeed, technical analysis yields slow information revelation from prices. The same lesson applies to price discovery mechanisms.

9.2 Strategic Trading with Long-lived Information

In this section we consider:

- Dynamic trading by a risk neutral large informed trader (“insider”) facing noise traders and risk neutral market makers (Kyle (1985)).
- Several extensions of the previous model to multiple informed traders, risk aversion, the effects of compulsory disclosure of trades by the insider, and a connection with the Glosten-Milgrom model.



9.2 Strategic Trading with Long-lived Information

9.2.1 The Kyle (1985) Model

Kyle (1985) considers a model where a large trader receives information and trades for T periods with a competitive risk neutral market making sector and noise traders:

- There is a single risky asset, with random (ex post) liquidation value θ , and a riskless asset, with unitary return, traded among noise traders and a large risk neutral informed trader (the “insider”).
- The insider observes θ , and the competitive market making sector intermediates trading.
- The informed trader acts strategically, taking into account the effect his demand has on prices, and faces a trade-off:
 - Taking positions early, and increasing profits then, leaks information to the market and diminishes profits later.

9.2 Strategic Trading with Long-lived Information

9.2.1 The Kyle (1985) Model

Consider period t

- The insider's information is given by $\{\theta, p^{t-1}\}$.
- He submits a market order: $\Delta Y_t(\theta, p^{t-1})$.
- Noise traders submit the aggregate order u_t and the order flow is then $\omega_t = \Delta y_t + u_t$.
- Competitive risk neutral market makers set prices efficiently conditional on the observation of the order flow: $p_t = E[\theta | \omega^t]$.
- All random variables are normally distributed with the sequence $\{u_t\}$ i.i.d. $N(0, \sigma_u^2)$. The liquidation value θ and the sequence $\{u_t\}$ are mutually independent.
- The profits of the insider due to his period t 's trade are $\pi_t = (\theta - p_t)\Delta y_t$.
- The profits of the insider on trades from period t to T are $\pi_t^T = \sum_{k=t}^T \pi_k$. His initial wealth is normalized to zero.

9.2 Strategic Trading with Long-lived Information

9.2.1 The Kyle (1985) Model

Kyle (1985)

Proposition

There is a unique linear equilibrium for $t = 1, 2, \dots, T$. It is given by:

$$\Delta Y_t(\theta, p^{t-1}) = \alpha_t(\theta - p_{t-1}),$$

$$E[\pi_t^T | \theta, p^{t-1}] = h_{t-1}(\theta - p_{t-1})^2 + \delta_{t-1}, \text{ and } p_t = \lambda\omega_t + p_{t-1},$$

where $p_0 \equiv \bar{\theta}$, $\omega_t = \Delta y_t + u_t$, $\lambda_t = \tau_u \alpha_t / \tau_t$, and $\tau_t = \tau_\theta + \tau_u \sum_{k=1}^t \alpha_k^2$. The constants α_t , h_t and δ_t are the unique solution to the difference equation system

$$h_{t-1} = \frac{1}{4\lambda_t(1 - \lambda_t h_t)}, \quad \alpha_t = \frac{1 - 2\lambda_t h_t}{2\lambda_t(1 - \lambda_t h_t)}, \quad \delta_{t-1} = \delta_t + h_t \lambda_t^2 \tau_u^{-1},$$

subject to the boundary conditions $h_T = 0$, $\delta_T = 0$, and the second order conditions $\lambda_t(1 - \lambda_t h_t) > 0$ for $t = 1, 2, \dots, T$.

9.2 Strategic Trading with Long-lived Information

9.2.1 The Kyle (1985) Model

Proof (Outline)

- 1 The strategy of the insider and its expected profits are obtained as a function of the market depth parameters λ_t at a linear equilibrium. Competitive market making at a linear equilibrium yields a price process of the form as in the proposition.
- 2 The boundary conditions $h_T = 0, \delta_T = 0$ imply that no profits are to be made after trade is completed. In the last period the trading intensity, as in the static model fulfills $\alpha_T \lambda_T = 1/2$.
- 3 The form of the strategy is also as in the static model with p_{T-1} taking the role of $\bar{\theta}$: $\Delta Y_T(\theta, p^{T-1}) = \alpha_T(\theta - p_{T-1})$. Indeed, competitive market making implies $p_{T-1} = E[\theta | p^{T-1}] = E[\theta | p_{T-1}]$
- 4 The properties of the price process yield immediately a quadratic value function of the form

$$E[\pi_t | \theta, p^{T-1}] = h_{T-1}(\theta - p_{T-1})^2, \text{ with } h_{T-1} = \alpha_T(1 - \alpha_T \lambda_T) = \frac{\lambda_T}{4}.$$

Using an induction argument the recursive form of the value function follows as stated in the proposition.

9.2 Strategic Trading with Long-lived Information

9.2.1 The Kyle (1985) Model

- 5 For any $t < T$, $\alpha_t \lambda_t < 1/2$: the insider considers the future information leakage of the impact of his trades.
- 6 From $\alpha_t = (1 - 2\lambda_t h_t)/(2\lambda_t(1 - \lambda_t h_t))$, and $\lambda_t = \tau_u \alpha_t / \tau_u$ we can obtain the cubic equation in λ_t , which has three real roots, the middle one satisfying the second order condition.
- 7 The difference equation system can be iterated backwards for a given τ_T (recall that $h_T = 0$). Only one terminal value is consistent with the prior.

9.2 Strategic Trading with Long-lived Information

9.2.1 The Kyle (1985) Model

At equilibrium:

- Due to risk neutral, competitive market makers the price is a sufficient statistic for public information $\omega^t = \{\omega_1, \omega_2, \dots, \omega_t\}$.
- The insider buys or sells in period t according to whether the liquidation value is larger or smaller than public information p_{t-1} .
- Information is gradually incorporated into the price, as an outcome of the trade-off faced by the insider, as price precision increases with t but remains bounded.
- The insider has **no** incentive to introduce noise in his order:
 - As in the static model he is optimizing at any t against a fixed conjecture on the behavior of market makers: λ_t .
 - For a given λ_t it is optimal **not to** introduce noise in the order since the only effect of placing a noisy order is just to distort trade from its optimal level given θ .

Things change if the informed trader is forced to disclose his trade at the close of the period.

9.2 Strategic Trading with Long-lived Information

9.2.1 The Kyle (1985) Model

- Kyle (1985) analyzes a continuous time version letting the intervals between trades tend to 0.
 - Noise trading, as well as equilibrium prices, follow then a Brownian motion.
 - Market depth is constant over time, information is incorporated into prices at a constant rate with all information incorporated at the end of trading. Prices converge to θ as the end of the horizon approaches.
- Back (1992) extends the model to continuous time. Back and Pedersen (1998) consider the case in which the monopolistic insider receives a flow of private information on top of an initial stock of information.
- Chau and Vayanos (2007) consider a steady state infinite horizon model in which the insider receives information every period about the expected growth rate of asset dividends.

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

Competition among insiders

Holden and Subrahmanyam (1992)

- Several insiders, all observing the fundamental value.
- Information is incorporated into prices much more quickly: all information is incorporated immediately as the interval between auctions tends to zero.
- Reason: equally informed agents trade more aggressively (Holden and Subrahmanyam (1994)).
- If the insiders are risk averse information revelation further speeds up, with a resulting increasing pattern of market depth.
- Reason: risk averse traders want to trade early to avoid future price uncertainty.

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

Foster and Viswanathan (1996)

- Several risk neutral informed agents each receiving a noisy signal of the fundamental value.
- The information structure is symmetric and error terms in the signals are potentially correlated: both the cases of all insiders receiving the same signal and receiving (conditionally) independent signals are covered.
- Focus: linear recursive Markov perfect equilibria and show that the problem of forecasting the forecast of others (infinite regress) does not arise in equilibrium, neither with one player deviations in order to check for equilibrium, because a sufficient statistic for the past can be found.
- The latter is a consequence of the combination of the recursive structure of the model, normality and competitive market making (much as in Vives (1995)).

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

- Simulations show that
 - ① Prices are less revealing the lower the correlation of private signals.
 - ② The correlation of private signals conditional on public information decreases over time and becomes negative towards the end of the horizon with enough trading rounds.
- Reason:
 - ① The more similar the information traders have, the more they compete and more of their information is transmitted to prices.
 - ② The competitive market making sector is learning basically the average of the signals of the traders and this means that the covariance between individual signals conditional on public information, when close to the average signal, must be negative.
- An informed trader will learn faster from the order flow than the market maker and this means that by trading aggressively he will reveal more to the competitors than to the market makers.

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

- Therefore, informed traders will be cautious and play a waiting game trying to induce the competitor to reveal information.
- Contrast with Holden and Subrahmanyam (1994): informed traders do not learn anything from each other as they receive the same signal.

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

Disclosure of trades and dissimulation strategies

Huddart, Hughes and Levine (2001)

- Kyle model where the insider has to disclose his trade before the next round of trading (e.g., US regulation).
- The equilibrium strategy of the insider in the Kyle (1985) model is no longer optimal:
 - 1 After the first round of trade it would be fully revealing of θ .
 - 2 This would induce the competitive market making sector to let depth be infinite in the second period and the insider would have the opportunity to make unbounded profits.
- The insider has to dissimulate his trade by introducing noise in his order.

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

- This noise is uncorrelated with all other random variables in the model, with mean zero and variance $\sigma_{\eta_t}^2$.
- The parameters of the randomization are not observable by market makers. The linear equilibrium for $t = 1, 2, \dots, T$:

$$\begin{aligned}\Delta y_t &= \alpha_t(\theta - E[\theta|\Delta y^{t-1}]) + \eta_t, \\ p_t &= \lambda_t \omega_t + E[\theta|\Delta y^{t-1}],\end{aligned}$$

with

$$\begin{aligned}\omega_t &= \Delta y_t + u_t, \alpha_t = \frac{1}{2\lambda_t(T-t+1)}, \\ \lambda_t &= \sqrt{\frac{\tau_u}{4T\tau_\theta}}, \text{ and } \sigma_\eta^2 = \frac{\sigma_u^2(T-t)}{T-t+1}.\end{aligned}$$

- Furthermore,

$$E[\theta|\Delta y^t] = E[\theta|\Delta y^{t-1}] + 2\lambda_t \Delta y_t, \text{ and } \text{Var}[\theta|\Delta y^t] = \frac{\sigma_\theta^2(T-t)}{T}.$$

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

Equilibrium properties:

- The dissimulation strategy of the insider involves setting
 - ① In every period the variance of added noise in his trade equal to the variance of the information-based component:

$$\sigma_{\eta_t}^2 = \alpha_t^2 \text{Var}[\theta - E[\theta | \Delta y^{t-1}]].$$

- ② The total variance of his trade equal to the variance of noise trading:

$$\text{Var}[\Delta y_t] = \sigma_u^2.$$

- The second part of the strategy camouflages the insider behind the noise traders and the first part makes difficult to distinguish between information-based and random-based trades once they are disclosed.
- The conditional volatility of θ at the start of period t , $\text{Var}[\theta | \Delta y^{t-1}]$, is smaller than with no disclosure and its reduction over time is constant across periods.
- The trading intensity of the insider is increasing over time and the market depth parameter is constant over time (necessary to sustain the mixed strategy equilibrium).

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

Note that:

- In contrast, in the Kyle (1985) discrete time model λ_t is **decreasing** with t as more information is incorporated into the price.
- Depth is always larger with disclosure.
 - The reason is that with disclosure some of the trades of the insider are **not** information-based but randomly generated.
- Expected per period profits for the insider are constant over time

$$\lambda_t \sigma_u^2 = \frac{1}{\sqrt{4T\tau_u\tau_\theta}}$$

and are lower round by round than with no disclosure (where they decline over time).

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

Risk averse traders

Guo and Kyle (2005)

- Continuous time model over an infinite horizon with a risk averse informed trader who receives continuously new information about the dividend process, noise traders, and risk averse market makers.
- The informed trader and the market makers have CARA utility and the informed faces quadratic trading costs.
- Characterize linear Bayesian equilibria and use the results to explain financial anomalies. The key is the presence of risk averse market makers who ask compensation for bearing risk. This explains “excess volatility.”
- The model can explain also the momentum and reversal puzzles. Stock returns tend to show positive short-term autocorrelation (momentum) but negative long-term autocorrelation (reversal).

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

The explanation is as follows:

- The orders from the informed trader are positively autocorrelated in the short run because he wants to smooth his trade over time in order to minimize trading and market impact costs. This may dominate the negative autocorrelation of orders from liquidity traders.
- Since the informed trader is risk averse, his order is negatively related to his inventory of the stock and private information is mean-reverting, his position will be mean-reverting in the long run. Together with the mean reverting position of liquidity traders this explains the long run negative autocorrelation in stock returns.
- Model provides an explanation of the anomalies that does not rely on the irrationality of traders.
- Alternative behavioral explanation: Daniel, Hirshleifer and Subrahmanyam (1998) based on the overconfidence of investors about the precision of their information and biased self-attribution.
- Other behavioral explanations of the anomalies have been provided by Barberis, Shleifer and Vishny (1998) and Hong and Stein (1999).

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

Kyle meets Glosten and Milgrom

Back and Baruch (2004)

- Extend Glosten and Milgrom (1985) to consider a single informed trader who uses market orders and decides, in continuous time, the optimal trading times.
- Uninformed buy and sell orders arrive as a Poisson process with constant and exogenous arrival intensities.
- Market makers are competitive and risk neutral, post bid and ask prices and see individual trades.
- The authors show that if the liquidation value of the risky asset follows a Bernoulli distribution and the informed trader knows the liquidation value, there is an equilibrium in which he follows a mixed strategy between trading and waiting.
- This means that both informed and uninformed traders arrive stochastically from the perspective of market makers, as assumed in the Glosten and Milgrom model.
- The equilibrium in this Glosten and Milgrom model is close to the equilibrium in the continuous-time version of the Kyle (1985) model when uninformed traders arrive frequently and trade size is small.

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

Parallel to the gradual trade of the insider in Kyle (1985) is the probabilistic waiting to trade of the informed trader:

- It is shown also that the bid-ask spread is almost “twice lambda” times the order size in the Kyle-type model and that the informed trader, in some circumstances, may randomize over trades that go against his information.
- Relevance: this shows that the more tractable Kyle (1985) model (with discrete batch auctions) is consistent also with the more common case where market makers set bid-ask prices and see individual trades.

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

- Back and Baruch (2007) generalize the model to encompass multiple order sizes and limit order markets. Both the informed trader and discretionary liquidity traders submit market orders and choose between block orders or a series of small orders. Liquidity providers are competitive and risk neutral.
 - Aim: compare floor exchanges, where a uniform price is established and an open limit order book, where there is discriminatory pricing.
 - In the first case prices are the expectation of the fundamental value conditional on public information and order size; in the second case, ask (bid) prices are “upper (lower) tail” expectations of the fundamental value.

9.2 Strategic Trading with Long-lived Information

9.2.2 Extensions

The model allows for larger traders to work their orders and pool with small traders.

- In a floor exchange it is never an equilibrium for all traders to use block orders: any equilibrium must involve at least partial pooling.
- If traders can submit orders an instant apart – effectively with no execution difference from a block trade – then the block-order equilibrium in the limit-order market is equivalent to a fully pooling worked-order equilibrium on the floor exchange.
- The incentive for large traders to pool with small ones is that if they do not, since small orders supposedly have a lessened adverse selection problem, prices for small orders would be more favorable.
- However, in a pooling equilibrium in the floor exchange, market makers do not know whether after an order there will be more from the same trader in the same direction.
- Thus, ask prices will also be upper-tail expectations as in the limit order market.
- The authors claim that their floor exchange model is a good representation of trade in the CBOE and the hybrid design in the NYSE.

9.2 Strategic Trading with Long-lived Information

9.2.3 Summary

Main learning points of the section:

- A large informed trader (an “insider”) has incentives to trade slowly so as not to reveal too much information and keep an informational advantage over uninformed traders and market makers. As the number of trading rounds increases information is incorporated in the price at a constant rate and risk neutral market makers keep a constant market depth.
- The insider will try to camouflage behind liquidity traders but has no incentive to introduce noise in his order to confuse market makers.
- Competition among strategic informed traders speeds up information revelation when they have symmetric information; otherwise informed traders may play a waiting game trying to induce the competitor to reveal information.
- If an insider has to disclose his trades then he does have an incentive to dissimulate his trades by randomizing optimizing his camouflage behind liquidity traders and obscuring the separation of information-based from liquidity-based trades. Disclosure increases market depth and information revelation, and decreases the expected profits of the insider.

9.2 Strategic Trading with Long-lived Information

9.2.3 Summary

- Financial market anomalies, such as the momentum and reversal in stock returns, can be explained with rational risk averse traders.
- Quote-driven markets (like in Glosten and Milgrom (1985)) and order-driven markets (like in Kyle (1985)) have a close connection when in the former uninformed traders arrive frequently and the trade size is small. This has important implications for the equivalence of trading in a floor exchange as compared to trading in a limit-order market, and for hybrid markets as well.

9.3 Market Manipulation and Price Discovery

We have seen how an insider may have incentives to dissimulate trades if he has to disclose them before trading again.

- A distinct possibility is market manipulation: an agent takes covert actions attempting to change the terms of trade in his favor.
- We provide first a quick survey of the literature on the topic, and analyze the possibilities of manipulation in the price discovery process studied before.



9.3 Market Manipulation and Price Discovery

9.3.1 Market Manipulation in the Literature

The stock-price manipulation literature can be classified according to whether manipulation is based on:

- 1 Actions that change the perceived value of the asset (e.g. Vila (1989)).
- 2 Releasing misleading information (Vila (1989) and Benabou and Laroque (1992)).
- 3 Purely based on trade (Hart (1977), Jarrow (1992), Allen and Gale (1992), Allen and Gorton (1992), Kumar and Seppi (1992), Fishman and Hagerty (1995), and Chakraborty and Yilmaz (2004)).

And according to whether the trader that manipulates the market is

- 1 Informed or
- 2 Uninformed

9.3 Market Manipulation and Price Discovery

9.3.1 Market Manipulation in the Literature

Examples:

- Allen and Gorton (1992): explain price manipulation by an uninformed agent in the presence of asymmetries in noise trading (noise selling is more likely than noise buying) or asymmetries in whether buyers or sellers are informed (with short-sale constraints to exploit good news is easier than to exploit bad news).
- Allen and Gale (1992)/Fishman and Hagerty (1995): an uninformed trader can pretend to be informed to manipulate the price and make money. In Fishman and Hagerty (1995) uninformed "insiders" may exploit the inability of market makers to distinguish trades of uninformed agents from those of insiders with private information.
- Goldstein and Guembel (2007) show how the allocation role of prices opens the possibility of market manipulation.

9.3 Market Manipulation and Price Discovery

9.3.1 Market Manipulation in the Literature

Contrarian behavior, or trade against one's information, is obtained in some instances in the literature.

- John and Narayanan (1997): variation of Fishman and Hagerty (1995) in which mandatory disclosure of trades of corporate insiders gives them incentives to manipulate the market using a contrarian strategy.
- The insider trades in a first period against his information to unwind his position in a second period.
- Foster and Viswanantan (1994): example of a duopoly where information has a common and a private component and where the better informed agent tries to minimize the learning of the lesser informed one.
 - This market manipulation may lead to contrarian behavior by the better informed trader if the private and common signals have very disparate realizations (something that happens with low probability).
- Chakraborty and Yilmaz (2004) find that insiders may trade in the wrong direction when there is uncertainty about their presence in the market and there is a large number of periods before information is revealed.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

Medrano and Vives (2001): Price manipulation by a strategic trader in a price discovery mechanism used in opening auctions.

- Consider the price discovery process with a random opening time for the market studied before with
 - ① Competitive informed traders, with mass $1 - \mu$ each with constant degree of risk aversion ρ .
 - ② Competitive risk neutral market makers.
 - ③ Noise traders.
 - ④ A large risk neutral informed trader, with mass μ who knows the liquidation value θ .
- This model belongs to the third class of manipulation models with trade-based manipulation.
- In our case the objective of the “insider” is to neutralize the informative trades that competitive informed agents make.
- In order to this the insider will use a contrarian strategy. Introducing a random opening time, like in Xetra, limits **but does not eliminate** the incentives to manipulate the market.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

Model

- The insider's information set at round t is given by $\{\theta, p^{t-1}\}$, where $p^{t-1} = \{p_1, p_2, \dots, p_{t-1}\}$.
- Denote his desired position at t by y_t .
- The information set of competitive informed trader i is given by $\{s_i, p^{t-1}\}$, where s_i is his private signal about θ , and his (symmetric) market order of the type $X_t(s_i, p^{t-1})$.
- Noise traders submit the aggregate order u_t .
- All random variables are assumed to be normally distributed.
- As usual we make the convention that

$$\tilde{s} = \frac{1}{1 - \mu} \int_{1-\mu}^1 s_i di = \theta, \text{ a.s..}$$

- Insider as emerging from a coalition of small informed traders (of measure μ) who decide to form a cartel of investors and pool their information.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

- The order flow is

$$\omega_t = \mu y_t + \int_{\mu}^1 X_t(s_i, p^{t-1}) di + u_t.$$

- Competitive market makers set

$$p_t = E[\theta | \omega^t], \text{ where } \omega^t = \{\omega_1, \omega_2, \dots, \omega_t\}.$$

- If the market opens at stage t , θ is realized, trade occurs and this is the end of the story.
- Otherwise the tâtonnement continues.
- All trades are notional until the market opens. Informed traders can revise their orders before the market opens and they will have incentives to do so once they receive more public information since this helps them to predict better the net value $\theta - p_t$.
- The insider may be able to manipulate the information contained in prices and may have incentives to do so.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

At stage t

- A strategy for the insider is a market order $Y_t(\theta, p_{t-1})$ given that p_{t-1} is a sufficient statistic for past public information.
- The asset will be liquidated and trade realized in the period with probability γ_t . With the complementary probability the preopening period continues. Thus the insider obtains in expectation

$$E[\pi_t | \theta, p_{t-1}] = \gamma_t E[(\theta - p_t) \mu y_t | \theta, p_{t-1}] + (1 - \gamma_t) E[\pi_{t+1} | \theta, p_{t-1}].$$

- In a linear equilibrium competitive traders, given their preferences and symmetric information structure, will use a symmetric strategy.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

Medrano and Vives (2001):

Proposition

Linear equilibria are characterized as follows for $t = 1, 2, \dots, T$:

$$Y_t(\theta, p_{t-1}) = \alpha_t(\theta - p_{t-1}), \quad X_t(s_i, p_{t-1}) = a_t(s_i - p_{t-1}),$$

$$p_t = \lambda_t \omega_t + p_{t-1}, \quad \omega_t = A_t(\theta - p_{t-1}) + u_t,$$

where $p_o = \bar{\theta}$, $\lambda_t = A_t \tau_u / \tau_t$, $A_t \equiv \mu \alpha_t + (1 - \mu) a_t$, and $\tau_t = \tau_\theta + \tau_u \sum_{k=1}^t A_k^2$. *At stage t the insider's expected continuation profit is given by*

$$E[\pi_{t+1} | \theta, p_{t-1}] = \mu(h_t(\theta - p_t)^2 + \delta_t).$$

The constants a_t, α_t, h_t , and δ_t are the solution to a system of difference equations for $t = 1, 2, \dots, T$.

and

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

Corollary

At a linear equilibrium the following inequalities hold for any t :

$$0 < a_t < \tau_\epsilon / \rho, \quad A_t > 0, \quad \lambda_t > 0, \quad 0 < \mu \lambda_t h_t < \frac{\gamma_t}{1 - \gamma_t},$$

$$0 < (1 - (1 - \mu)\lambda_t a_t) < 1, \quad 0 < 1 - \lambda_t A_t < 1.$$

- Existence is hard to prove. However, it can be proved in some special cases: $\mu = 0$ (as in Vives 1995), $\mu = 1$ (as Kyle 1985) or for $T = 2$ and γ_1 close to 0.
- With limit orders the dynamics are simplified and existence and uniqueness can be proved.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

Expected total volume at time t is given by

$$E[TV_t] = \frac{1}{2} ((1 - \mu)E[|X_t(s_i, p_{t-1})|] + \mu E[|Y_t(\theta, p_{t-1})|] + E[|\omega_t|] + E[|u_t|]),$$

where

$$E[|X_t(s_i, p_{t-1})|] = \left(\frac{2}{\pi}\right)^{1/2} a_t(\tau_\epsilon^{-1} + \tau_{t-1}^{-1})^{1/2},$$
$$E[|Y_t(\theta, p_{t-1})|] = \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{\alpha_t^2}{\tau_{t-1}}\right)^{1/2},$$
$$E[|\omega_t|] = \left(\frac{2}{\pi}\right)^{1/2} \left(\sigma_u^2 + \frac{A_t^2}{\tau_{t-1}}\right)^{1/2}.$$

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

When $\mu = 1$

- There is a monopolistic insider and it is possible to show the existence of a unique linear equilibrium.
- The insider faces a more stark version of the insider's trade-off in the Kyle model.
 - At stage t his future profit will decrease by placing a market order if there is no trade, because of the information leaked to the market makers.
 - By not submitting an order if trade occurs, his future profit will be zero because θ will have been revealed.
 - The optimal market order, which balances the two effects, implies a trading intensity that is lower than in the one-shot model where there is trading with probability one.
- In our monopolistic market $\lambda_t \alpha_t < 1/2$, for all $t < T$ and $\lambda_T \alpha_T = 1/2$ (since for $t = T$ the model becomes like the static Kyle model).

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

- The large informed trader refrains from trading too aggressively because there is a positive probability that there is no trade.
- This suggests that his trading intensity should be increasing in the probability γ_t (and this is confirmed by the simulations).
- For the central case where $\gamma_t = \gamma^{T-t}$, and in contrast to the competitive economy (where $\mu = 0$), no matter how long the horizon is the price precision is **bounded above** (and the bound depends only on the parameter γ).
- The monopolistic insider prevents the full revelation of θ no matter how many rounds the tâtonnement has.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

Simulation analysis (assuming that $\gamma_t = \gamma^{T-t}$):

- The responsiveness to private information increases monotonically with t ; in the monopolistic case at an accelerating rate and in the competitive case at a decelerating rate.
- The informativeness of prices τ_t increases monotonically with t ; in the monopolistic case at an accelerating rate close to the opening and in the competitive case at the rate of t .
- Market depth $(\lambda_t)^{-1}$ tends to infinity at a rate of t in the competitive equilibrium; in the monopolistic equilibrium, in general, it decreases during the first rounds of the tâtonnement and then increases as the probability that there will be trade tends to one.
- The unconditional volatility of prices $\text{Var}[p_t]$ increases monotonically towards τ_θ^2 in both cases. However, in the competitive economy it gets close to τ_θ^2 in the first few rounds of tâtonnement while it is close to zero in the monopolistic economy (because market depth is extremely high).
- Expected trading volume: In the competitive economy, the expected volume traded by informed agents is decreasing for t large, while in the monopolistic economy it increases monotonically.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

For the general case:

$$A_t = \mu\alpha_t + (1 - \mu)a_t, \tau_t = \tau_{t-1} + \tau_u A_t^2, \text{ and } \lambda_t = \frac{A_t \tau_u}{\tau_t}.$$

Therefore, both the information of competitive informed traders and that of the insider contribute to form

- price informativeness τ_t and
- depth λ_t^{-1} .

If γ_t is small, the insider “manipulates” the market setting $\alpha_t < 0$ in order to decrease price informativeness:

- If $\gamma_t = 0$ there is no danger that the market opens and $\alpha_t = -(1 - \mu)a_t/\mu$ so that $A_t = 0$ and informed traders’ information is completely neutralized by the insider.
- If $0 < \gamma_t < 1$, the insider must balance reducing the informativeness of prices by choosing a low (and possibly negative) α_t , and trading intensely (choosing α_t close to the static equilibrium value) to obtain a high profit if trades are executed.
- If $\gamma_t = 1$, the insider behaves as in the static version of the model.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

The simulations performed for the case $\gamma_t = \gamma^{T-t}$ corroborate the analysis and conjectures above:

- α_t is increasing in t and in γ_t .
- τ_t is strictly convex in t .
- For T large enough:
 - ① $\alpha_t < 0$ for t low.
 - ② $\text{Var}[p_t|p_{t-1}]$ may be hump-shaped or increasing and U-shaped, or decreasing in t .
 - ③ Total expected trading volume is U-shaped in t .
- Further simulations support the conjecture that for $\mu > 0$, as in the case of a monopolistic insider, for any given γ there is an upper bound for the price precision, no matter the length T of the horizon.
- A larger size of the insider implies a lower limit value for the price precision and this limit is attained in fewer rounds of trade. Indeed, when μ increases the average responsiveness to information tends to decrease and this impacts negatively on the informativeness of prices. A larger insider tends decrease the price precision and the expected volume traded.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

According to the simulations:

- The insider manipulates the market at the beginning of the price adjustment process (result (1)).
- As a consequence, the informativeness of prices is very low during the first stages and increases quite fast as t gets close to T .
- Result (3) is driven by the fact that the insider's expected trading volume is U-shaped. The expected volume traded by informed traders (ignoring the volume traded among competitive informed agents) equals

$$(\text{Var}[\theta|p_t])^{1/2} (\mu|\alpha_t| + (1 - \mu)a_t).$$

For γ not too high this volume will have a U-shaped temporal pattern because $|\alpha_t|$ does and dominates. This in turn dominates the decreasing tendency of $\text{Var}[\theta|p_t]$.

- The general pattern of results obtained hold also in the case that the strategic and the competitive informed agents use demand schedules instead of market orders.

9.3 Market Manipulation and Price Discovery

9.3.2 Strategic Behavior and Price Discovery

The interaction between a strategic informed trader and a sector of competitive informed agents in the model presented in this section yields outcomes consistent with the empirical evidence available from the Paris Bourse: Biais, Hillion and Spatt (1999) find that

- The last 15 minutes before the opening are the most active order placement period in the day.
- The average size of orders placed in the preopening period increases as we get closer to the opening and large traders place sometimes not aggressive orders and tend to modify their orders.
- The volume of trade has typically a U-shaped form dropping after the first round to increase sharply later when approaching the opening.
- The hypothesis of semi-strong efficiency for prices close to the opening cannot be rejected.
- The speed of learning from prices is of the order of $t^{3/2}$, in the second part of the preopening, where t is the number of rounds in the tâtonnement: the precision of prices grows more than linearly towards the end of the process. This speed of learning is easy to generate in the theoretical model.

9.3 Market Manipulation and Price Discovery

9.3.3 Summary

- Market manipulation is a distinct possibility when there are large traders in the market.
- In the preopening period of a price discovery process we have seen how a strategic informed trader has incentives to use a contrarian strategy to suppress the information leakage from the price deriving from the competitive behavior of other informed traders.
- This manipulation is understood by everyone in the market to happen in equilibrium.

9.4 Strategic Trading with Short-lived Information

- Up to now we have studied the impact of long-lived information on price informativeness, volatility and volume in the presence of strategic traders and noise traders which were given no choice of when to trade.
- However, a strategic trader may possess also short-lived information and “noise” traders may have at least some discretion about when to trade.
- Admati and Pfleiderer (1988) use a dynamic model with T periods where the information of n insiders is short-lived and where there are some liquidity traders that can choose when to trade.
- This is made to explain the regularity that the average intraday volume and variance of price changes in the NYSE is U-shaped.
- The basic idea is that the intraday trading patterns for volume and price volatility may be explained by the incentives of liquidity and informed traders to cluster their trades.



9.4 Strategic Trading with Short-lived Information

Model

- The liquidation value of the single risky asset is given by $\theta = \bar{\theta} + \sum_{t=1}^T \theta_t$, where $\bar{\theta}$ is a known parameter, $\{\theta_t\}$ are independently normally distributed random variables with mean zero and variance σ_t^2 .
- There are n_t informed traders in period t and they all see the same signal about the innovation next period $s_t = \theta_{t+1} + \epsilon_t$, where ϵ_t is normally distributed with mean 0 and variance $\sigma_{\epsilon_t}^2$. Error terms in signals and innovations in the fundamental value are mutually independent.
- The value of θ_t becomes known at the beginning of period t and therefore the information of the informed traders is short lived.
- Denote by $Y_t(s_t)$ the market order of an informed trader in period t .

9.4 Strategic Trading with Short-lived Information

- There are two types of liquidity traders in period t :
 - The usual noise traders, trading according to a normal random variable u_t , and
 - m discretionary liquidity traders who can choose when to trade within a time interval (say the trading day).
- When discretionary liquidity trader j trades he has a demand z_j of shares that cannot be split during the trading period.
- His demand in period t is

$$z_{jt} = \begin{cases} z_t & \text{if he trades} \\ 0 & \text{otherwise.} \end{cases}$$

- All traders submit market orders and a competitive risk neutral market making sector sets prices upon observing the order flow and public information.

9.4 Strategic Trading with Short-lived Information

- Denote by y_{it} the order of informed trader i in period t . The order flow in period t is given by

$$\omega_t = \sum_{i=1}^{n_t} y_{it} + \sum_{j=1}^m z_{jt} + u_t.$$

- It is assumed that

$$\{z_1, z_2, \dots, z_m, u_1, u_2, \dots, u_{T-1}, \theta_1, \theta_2, \dots, \theta_T, \epsilon_1, \epsilon_2, \dots, \epsilon_{T-1}\},$$

are mutually independent normally distributed random variables.

9.4 Strategic Trading with Short-lived Information

- The competitive market making sector in period t sets prices conditional on the order flow ω_t and on public information $\theta^t \equiv \{\theta_1, \theta_2, \dots, \theta_t\}$,

$$\begin{aligned} p_t &= E[\theta | \omega_t, \theta^t] \\ &= E[\theta | \theta^t] + \lambda_t \omega_t \\ &= \bar{\theta} + \sum_{k=1}^t \theta_k + \lambda_t \omega_t. \end{aligned}$$

where $\lambda_t = \text{Cov}[\theta_{t+1}, \omega_t] / \text{Var}[\omega_t]$. The inverse of the parameter λ_t is as usual a measure of the depth of the market.

9.4 Strategic Trading with Short-lived Information

- In period t the informed traders compete taking into account the price rule of the market makers and trader i demands $y_{it} = \alpha_t s_t$. In equilibrium

$$\alpha_t = \sqrt{\frac{\Psi_t}{n_t \text{Var}[s_t]}}, \text{ where } \Psi_t = \text{Var} \left[\sum_{j=1}^m z_{jt} + u_t \right],$$

is the variance of total liquidity trading in the period, and

$$\lambda_t = \frac{\sigma_{t+1}^2}{n_t + 1} \sqrt{\frac{n_t}{\Psi_t \text{Var}[s_t]}}.$$

9.4 Strategic Trading with Short-lived Information

- Not surprisingly, an increase in total liquidity demand increases:
 - ① The sensitivity of informed traders to their information, as they can camouflage better behind noise traders.
 - ② Market depth, as the order flow becomes less informative.
- As the number of informed traders n_t increases, each one of them responds less to his information, since they all receive the same signal, and λ_t decreases, as they are not able to restrict their trade enough to control the information leakage in the order flow: the adverse selection problem of market makers is less severe because of competition among the informed.

9.4 Strategic Trading with Short-lived Information

- A discretionary liquidity trader chooses to trade when the cost of trading is lowest, that is, when his expected losses:

$$E[(p_t - \theta)z_j | \omega^{t-1}, \theta^t, z_j] = \lambda_t z_j^2, \text{ where } \omega^{t-1} = \{\omega_1, \omega_2, \dots, \omega_{t-1}\},$$

are minimal

- Therefore, liquidity traders would like to trade when λ_t is lowest and this happens when there are a **lot of other liquidity traders** (λ_t is decreasing in Ψ_t).
- It is not difficult to see that liquidity traders face a coordination problem and that there will exist multiple equilibria: If in period t there is a lot of discretionary liquidity trading then λ_t will be low and this will attract more liquidity traders.
- There is always an equilibrium where all discretionary liquidity trading happens in the same period and generically only this type of equilibrium is possible.
- Furthermore, insiders like to trade also in a deep market to disguise better their trades.

9.4 Strategic Trading with Short-lived Information

Concentration of discretionary liquidity trading in one period

- will induce more trading by informed traders and explains a peak in volume.
- In fact, this peak in volume occurs even if the rate at which information becomes public is constant (e.g. $\sigma_t^2 = 1$), signals have the same precision, $\tau_{\epsilon_t} = \tau_{\epsilon}$, and the amount of nondiscretionary noise trading is constant, $\text{Var}[u_t] = \sigma_u^2$, in any period.
- However, the model does not explain volatility changes. Indeed, if there are the same number of informed traders in any period, $n_t = n$ ($\tau_{\epsilon_t} = \tau_{\epsilon}$, $\sigma_t^2 = 1$) for all t , then the volatility of price changes is

$$\text{Var}[p_t - p_{t-1}] = \frac{\tau_{\epsilon}}{1 + \tau_{\epsilon}} \left(\frac{1}{\tau_{\epsilon}} + \frac{n}{1 + n} + \frac{1}{1 + n} \right) = 1.$$

- The informativeness of prices about the dividend innovation is

$$(\text{Var}[\theta_{t+1}|p_t])^{-1} = \left(1 + \frac{n\tau_{\epsilon}}{1 + n + \tau_{\epsilon}} \right)^{-1},$$

is also constant across time periods.

9.4 Strategic Trading with Short-lived Information

- All this holds irrespective of Ψ_t and therefore is true for the period in which liquidity trading is concentrated.
- Indeed, as in the Kyle (1985) static model the informativeness of prices, and therefore conditional volatility in a semi-strong efficient market, is independent of the amount of noise trading:
 - More liquidity trading entices risk neutral informed traders to trade more intensely and this. . .
 - is just sufficient to keep the order flow with the same information content.
- With no informed trading

$$p_t - p_{t-1} = \theta_t, \text{ and } \text{Var}[p_t - p_{t-1}] = 1$$

- When there is informed trading the result is the same. To generate more interesting volatility patterns we need to have different numbers of informed traders in different periods.

9.4 Strategic Trading with Short-lived Information

Endogenize the number of insiders in any period t studying the incentives to acquire information:

- Whenever the number of insiders is known in equilibrium concentrated trading patterns are reinforced.
- This is so because more liquidity trading:
 - ① Incentivates entry of informed traders and, because of enhanced competition among them, lowers trading costs for liquidity traders.
 - ② This reinforces the incentive of discretionary liquidity traders to trade in the period to start with.
- The expected profits of an informed trader in period t match the (share of) expected losses of liquidity traders:

$$\begin{aligned}\pi(n_t, \Psi_t) &= \frac{\lambda_t}{n_t} \Psi_t \\ &= \frac{1}{1 + n_t} \left(\frac{\tau_\epsilon \Psi_t}{(1 + \tau_\epsilon) n_t} \right)^{1/2},\end{aligned}$$

which are increasing in the total amount of liquidity trading in the period Ψ_t .

9.4 Strategic Trading with Short-lived Information

- At a free entry equilibrium where traders have to pay F to become informed

$$\pi(n_t, \Psi_t) \geq F,$$

and entry by one more informed trader would induce an equilibrium with negative net expected profits.

- Now with more information about the dividend process concentrated when liquidity trading is high we have that volatility and volume are positively correlated.
- In summary, when liquidity traders have discretion about when to trade they will tend to concentrate their trading in periods where market depth is high.
- This will become a self-reinforcing process where more liquidity traders and informed traders, with short-lived information, will do so also, generating volume and volatility peaks.

9.5 Strategic Hedging

So far we have seen models with

- Strategic behavior of traders privately informed about the fundamental value of the risky asset.
- The motivation for trade for those large traders is to exploit their information advantage.
- However, risk averse large traders may trade also for insurance motives when receiving a shock to their endowments or to hedge an investment.
- What are the consequences of strategic behavior when large traders trade for an insurance motive?



9.5 Strategic Hedging

Suppose each strategic trader only knows his endowment shock.

- What are the consequences for the speed of trading and welfare losses, as well as possibilities to manipulate the market?
- The case when the strategic trader has information about the fundamental value and wants to obtain insurance to hedge his investment has been examined in a previous section.
- Chau (2002) examines the dynamic incentives of a strategic trader to exploit his private information about the fundamental value as well as controlling his inventory after an endowment shock.
- We will display two patterns of trading by a large risk averse trader that suffers an endowment shock:
 - ① In the first we will see how private information about the endowment shock lead to slow trading and potentially large welfare losses.
 - ② In the second we will uncover an instance of market manipulation in the presence of noise traders.

9.5 Strategic Hedging

Vayanos (1999)

- Dynamic model with n CARA risk averse infinitely lived strategic agents who receive an endowment shock every period and want to insure against dividend risk.
- Dividends follow a random walk, dividend information is public, and the endowment shock to a trader is the only private information.
- Traders submit (continuous) demand schedules every period to a centralized market clearing mechanism and all random variables are normally distributed. There is no noise trading.
- The model may fit inter-dealer markets, where dealers want to share their inventory risk and participants are large. In the linear Nash equilibrium in demand functions studied prices are fully revealing of the endowment shocks because there is no noise trading.

9.5 Strategic Hedging

A first result: Agents trade **slowly** even when the time between trades tends to zero.

- Reason: making use of more trading opportunities makes price impact very important as a trade in one direction indicates many more trades in the same direction. To avoid that large agents trade slowly.
- The result goes away, and trade accelerates as the interval between trades decreases, as in the Coase conjecture, if endowment shocks are public information.
- According to the Coase conjecture a durable goods monopolist sells fast and prices converge to marginal cost fast as the interval between trades diminishes. Private information on endowments drives the slow trading result by increasing the price impact of a trade.

9.5 Strategic Hedging

- In equilibrium if a trader were to sell more shares than the other traders would incorrectly infer that he has received a larger endowment shock and expect more sales in the future.
- This does not happen when endowments are public information: With public information on endowments there is a continuum of equilibria because traders know the market clearing price and are indifferent about which demand function to submit for out of equilibrium prices.

9.5 Strategic Hedging

- Welfare loss due to strategic behavior:
 - ① Increases as the time between trades shrinks, in contrast to the public information case.
 - ② Is of the order of $1/n^2$ for a fixed length of the interval between trades (as in a k -double auction) but as this time interval tends to 0 the welfare loss is of the order of $1/n$.
- Dynamic trading with strategic behavior may imply a slower convergence to efficiency than static competition as the number of traders increases.

9.5 Strategic Hedging

- Vayanos (2001) studies a stationary model where one risk averse large trader receives a privately observed endowment shock every period and submits a market order to a competitive risk averse market making sector in the presence of noise traders.
- As before, information about asset payoffs is public and the large trader trades for an insurance motive.
- Results: After receiving an endowment shock the large trader reduces his risk exposure either by (1) selling at a decreasing rate over time or, more surprisingly, by (2) selling first to achieve optimal risk sharing and then engaging in a round trip transaction selling some more shares to buy them back later.

9.5 Strategic Hedging

- The second pattern, which happens when there is enough noise trading and the large trader is not very risk averse compared to market makers, has a manipulation flavor.
- Indeed, in the second pattern of trade market makers are misled by the first sale, thinking that has originated with the noise traders.
- However, the large trader knows that this is not the case and therefore that the price will fall. He exploits the situation by selling and buying back when the price has fallen. It is found also that when the time between trades tends to zero the information about the endowment of the large trader is reflected in the price very quickly.

9.5 Strategic Hedging

- In short, large risk averse traders when hedging endowment shocks which are private information will trade slowly, even when trade is very frequent.
- A consequence is that the welfare loss due to strategic behavior may increase as trade becomes more frequent and that as the number of traders increases an efficient outcome is approached more slowly than with one shot trading.
- Furthermore, large risk averse traders, when hedging endowment shocks which are private information, may have incentives to manipulate the market trying to mislead market makers, as in the case with privileged information about the fundamental value.

Summary

The chapter has examined the dynamics of competitive (mostly market order) markets allowing for strategic behavior.

- Learning from past prices (technical analysis) may be very slow, as in the canonical model of learning from others, if market depth is fixed exogenously.
- If competitive risk neutral market makers set the depth of the market then informational cascades are not possible and learning from prices in price discovery processes is faster.
- A large informed trader has incentives to trade slowly so as not to reveal too much information and keep an informational advantage over uninformed traders and market makers. The insider will try to camouflage behind liquidity traders but has no incentive to introduce noise in his order to confuse market makers.
- Competition among strategic informed traders speeds up information revelation when they have symmetric information; otherwise informed traders may play a waiting game trying to induce the competitor to reveal information.

Summary

- If an insider has to disclose his trades ex post then he will dissimulate by randomizing to optimize his camouflage behind liquidity traders and obscure the separation of information-based from liquidity trades. Disclosure increases market depth and information revelation, and decreases the expected profits of the insider.
- Market manipulation is a distinct possibility when there are large traders in the market. A strategic informed trader may have incentives to use a contrarian strategy to suppress the information leakage from the price deriving from the competitive behavior of other informed traders.
- When liquidity traders have discretion about when to trade they will tend to concentrate their trading in periods where market depth is high and this will become a self-reinforcing process where informed traders, with short-lived information, as well as more liquidity traders will do so also, generating volume and volatility peaks.
- Large risk averse traders hedging endowment shocks which are private information will trade slowly, even when trade is very frequent, and may have incentives to manipulate the market trying to mislead market makers.

Appendix

Proposition

With a competitive risk neutral market making sector there is a unique linear equilibrium. Traders use symmetric strategies

$$X_t(s_i, p^{t-1}) = a_t(s_i - p_{t-1}), \text{ with } a_t = \rho (\sigma_\epsilon^2 + \text{Var}[p_t|p_{t-1}])^{-1},$$

and prices are given by

$$p_t = \lambda_t \omega_t + p_{t-1}, \text{ with } \omega_t = a_t(\theta - p_{t-1}) + u_t,$$

$$\lambda_t = \tau_u a_t / \tau_t, \tau_t = \tau_\theta + \tau_u \sum_{k=1}^t a_k^2, \text{ and } \text{Var}[p_t|p_{t-1}] = \tau_{t-1}^{-1} - \tau_t^{-1}.$$

Proof

At a linear equilibrium and given our assumptions all random variables are normally distributed. At stage t maximization of a CARA utility function by trader i yields

$$X_t(s_i, p^{t-1}) = \frac{E[\theta - p_t | s_i, p^{t-1}]}{\rho \text{Var}[\theta - p_t | s_i, p^{t-1}]},$$

Appendix

where $p_t = E[\theta|\omega^t]$ from the competition among market makers.

- The expression is independent of i and therefore equilibria will be symmetric. We obtain that

$$p_t - p_{t-1} = \lambda_t \omega_t, \text{ where } \lambda_t = \tau_u a_t / \tau_t,$$

$$\omega_t = a_t(\theta - p_{t-1}) + u_t.$$

- Since $p_t = \lambda_t(a_t\theta + u_t) + (1 - \lambda_t a_t)p_{t-1}$, we have that

$$E[\theta - p_t | s_i, p^{t-1}] = (1 - \lambda_t a_t) \tau_\epsilon (\tau_{t-1} + \tau_\epsilon)^{-1} (s_i - p_{t-1}).$$

- Now, we have that

$$\begin{aligned} X_t(s_i, p_{t-1}) &= \frac{E[\theta - p_t | s_i, p_{t-1}]}{\rho \text{Var}[\theta - p_t | s_i, p_{t-1}]} \\ &= \frac{(1 - \lambda_t a_t) E[\theta - p_{t-1} | s_i, p_{t-1}]}{\rho((1 - \lambda_t a_t)^2 \text{Var}[\theta | s_i, p_{t-1}] + \lambda_t^2 \sigma_u^2)}. \end{aligned}$$

Appendix

- Using the expressions for conditional expectation and conditional variance we obtain:

$$\begin{aligned} a_t &= \rho (\sigma_\epsilon^2 + a_t \lambda_t \text{Var}[\theta | p_{t-1}])^{-1} \\ &= \rho (\sigma_\epsilon^2 + \text{Var}[p_t | p_{t-1}])^{-1}. \end{aligned}$$

- This yields the recursive cubic equation

$$F_t(a_t) = \left(\frac{\rho}{\tau_\epsilon} a_t - 1 \right) \tau_{t-1} + \rho \lambda_t a_t^2 = 0.$$

- It is clear that positive roots must lie in $(0, (\rho\sigma_\epsilon^2)^{-1})$. It can be easily checked that $F_t(0) < 0$, $F_t((\rho\sigma_\epsilon^2)^{-1}) > 0$, and that $F_t(a_t) = 0$ implies $F'_t(a_t) > 0$.
- It follows then that there is a unique positive root.

